

Index

- Accuracy. *See* Approximation; Error; Measurement
- Addition, low-stress, 220-26
- Additivity, 13-14, 20, 23-24, 25, 30
- Aids, teaching. *See* Manipulative devices
- Algorithms. *See* Low-stress algorithms
- Angle(s), 29-30, 72-73, 77, 154-55, 164
- Applications
 - of parallax, 141-42
 - in SI, 132-33
- Approximation
 - of area, 169-70
 - in experimental error, 176-79
 - in measurement, 84
- Area, 24
 - additivity property of, 25
 - approximating, by averaging-rectangle method, 169-73
 - comparison of, 26-27, 51, 75-76
 - congruence of, 15
 - determining irregular, 166-68
 - of a rectangle, 51-52
 - sources of error in finding, 106-8
 - spatial representation of, 46
- Area function, 24-27
- Attribute, measurable, 73-77, 88
- Behavior, measuring, 44
- Betweenness, on a line, 23
- Bisection of angle, 154-55
- Capacity. *See* Volume
- Carpenter's square, 152-55
- Cavalieri principle, 28
- Clinometer, angle, 164, 195-96
- Commensurability, 115
- Comparison
 - of angles, 33, 77
 - of areas, 26-27, 51, 75-76
 - of lengths, 36-37, 42, 45, 73
 - of times, 57, 58, 74-75
 - of volumes, 53, 76
 - of weights, 74
- Computation, 17, 124, 218-19. *See also* Low-stress algorithms
- Congruence property
 - of angle measure, 30
 - in area function, 25
 - in higher-dimensional space, 22
 - of length, 20
- Conservation
 - of area, 46-48
 - of energy, 175, 180, 183-84, 185
 - of liquid volume, 76-77
 - of momentum, 175, 180, 183, 185
 - of volume, 47-49, 53, 54
 - of weight, 55
- Constructions. *See* Models
- Coordinate system, for points on a line, 21
- Curriculum. *See also* Measurement program
 - effects of SI on, 134-36
 - place of low-stress algorithms in, 236-37
 - role of estimation in, 87-104 passim, 99-100
- Curve, length of, 24, 156-60
- Decimal notation, in SI, 128-29
- Derived scales, 31

- Developable surfaces, 160-61
- Distance, 20. *See also* Distance function
 - definition of, 21, 158
 - on a line, 21
 - on surfaces, 156
 - between two points, 15, 157-60
- Distance function
 - attributes of, as bases for transfer, 20
 - compared with area function, 25-27
- Division
 - facts, through low-stress addition, 225-26
 - of a line into equal parts, 153-54
 - low-stress, 233-36
- Error
 - accounting for, in measuring, 144, 146
 - anticipating sources of, 105, 108, 109-10, 114, 175-76
 - in approximating, 85
 - independent, 177-78
 - relative, 17
 - types of, 176
 - using graphs to evaluate, 94-96
- Estimating
 - definition of, 89
 - developing skill in, 93-94
 - problems in, 111-12
- Estimation
 - of area, 101
 - of length, 100
 - of mass, 101-2
 - of time, 101
 - of volume, 101
- Fractions, in SI, 129. *See also* Rational numbers
- Function
 - angle measure, 29-30
 - area, 24
 - volume, 27-29
- Geoboard, 76
- Geodesics, 156, 157-58, 158-60
- Graph, 73, 94-96
- Histogram, 177
- Homomorphism, 32, 33
- Incommensurability, 26
- Indirect measure, 30, 31
- Individualized instruction, 67-68, 76
- International System of Units. *See* SI
- Length
 - additivity property of, 20, 24
 - Archimedean property of, 20
 - comparison of, 36-37, 42, 73
 - congruence property of, 20
 - of a curve, 24
 - relations, transitivity of, 39-42
- Line
 - ability to represent, 38-39
 - betweenness on, 23
 - distance on, 21
 - division of, into equal parts, 153-54
- Linear measurement, unit of, 42
- Low-stress algorithms, 219-39
 - addition, 220-26
 - division, 233-36
 - multiplication, 226-31
 - subtraction, 231-33
- Manipulative devices, 187-209 *passim*
- Mass. *See* Weight
- Materials. *See* Manipulative devices
- Mathematics program. *See* Curriculum
- Measure. *See also* Measurement; Measuring
 - definitions of, 18, 88
 - of distance on a line, 21
 - as a function, 19
- Measurement. *See also* Measure; Measuring
 - accuracy in, 82
 - anticipating problems in, 114
 - appreciation for, 65
 - as approximation, 84-85
 - comparison in, 61, 62
 - definitions of, 18, 50, 88
 - error in, 16, 144, 146, 175-76
 - as a function, 89
 - importance of, 63-64
 - in modern technology, 1-10 *passim*
 - perception of, 61-62, 68-73
 - principles for effective teaching of, 61-64
 - skills necessary in, 61
 - steps in process of, 108-9
 - of time, 58
 - using the natural environment, 163-74
- Measurement program
 - content of, 66-67
 - objectives for, in elementary school, 61, 64-66
- Measure systems, 11, 63
 - derived, 31
 - as a model, 32, 133-34

Measuring. *See also* Measure;**Measurement**

angles, 29-30

definition of, 88

distances on surfaces, 156

irregular areas, 166-73

use of error in, 66

velocity of a projectile, 175-86

*passim***Metric materials, bibliography of,**

210-17

Metric space, 158**Metric system. *See* Measure; Measuring;****SI****Models**

building, 18

of measurement process, 106, 120

of measure systems, 32

Modular arithmetic, uses of, 127-28**Multiplication**

low-stress, 226-31

facts, through low-stress addition, 225

Nondevelopable surfaces, 161-62**Nonstandard measures, 77-79****Nuffield project, 97****Number line**

as basis for scales, 66

on instruments and scales, 84

Optical range finder, 142-44**Parallax**

activities with, 142-46

applications of, 138, 141-42

definition of, 138, 139

geocentric, 141

heliocentric, 141-42

Model T viewer, 144-46

Percent, in SI, 131**Perception**

of angles, 72

of area, 71

of length, activities for, 68

of temperature, 70

of time, 69

of volume, 71

of weight, 69

Perimeter, 22**Perpendicular bisector, 155****Photography, to measure velocity,**

147-51

Piaget, 38, 39, 43, 46, 47, 49, 50, 51,

52, 53, 55, 56, 57, 58, 67

Place value, metric, manipulative aids

in teaching, 208-9

Ratio, in SI, 130-31**Rational numbers, in SI, 131-32****Real numbers, in SI, 132****Referents, 97-98****Relations, in SI, 130-31****Relative error, 17****Scales, derived, 31****SI (Système International d'Unités)**

advantages of, 125

base units of, 80, 123

conventions of usage, 134

decimal notation in, 128-29

de-emphasis of computation in, 124

dominance of base ten in, 126-28

estimating in, 132

as a model for mathematical structure,
133-34

as a system of measurement, 63

as used to simplify teaching of
measurement, 60**Space**

congruence property in higher

dimensional, 22

measurement in exploration of, 1-2,
81

metric, 158

Spatial representation, 37-39, 46-49**Speed**

definition of, 56

fundamental to concept of time, 58

Sphere, volume of, 109-10**Standard deviation, 176-77, 178****Standard measures, 79-80. *See also* SI****Standard referents**

development of, 65

functions of, 63

Subtraction, low-stress, 231-33**Surface area of developables, 156****Système International d'Unités. *See* SI****Temperature, 70-71, 197-98****Tessellations. *See* Tiling****Tiling, 50-51**

approach to area, 26

Time

comparison of, 57, 58, 74-75

development of concept of, 56

length of, 56-57

measurement of, 58

speed as basis for concept of, 57

Transfer

designing instruction for, 12-34

passim

of estimation skills, 98

Transfer (*continued*)

- of learning, 12, 33-34
 - as primary goal of measure experiences, 12
 - of transitivity, 42
- Transitivity**, 36, 39-42 *passim*
- of length relations, 39-42
 - transfer of, 42
- Trilateration**, 166, 168

Unit(s)

- of area, 26, 51
- of area measurement, 49
- of linear measurement, 42
- in SI, 123
- of volume measurement, 49, 52

Vectors, addition of, 131

Vector spaces, in elementary school, 131

Velocity, 58. *See also* Speed; Time

- measuring, of a projectile, 175-86
- passim*
- using photography to measure, 147-51

Volume

- comparison of, 53, 76, 77
- concepts of, 47-49
- difference in liquid and solid, 28
- differentiating length and area, 28
- function, 27-29
- mathematical structure for measure
 - of, 27-29
- models in teaching, 46
- properties of, 27-28
- of a rectangular solid, 52-53
- sources of error in finding, 109-10
- spatial representation of, 46-49
- of a sphere, activity in finding, 109-10

Weight, 55, 206-7

